

МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ
ТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ
ГРАЖДАНСКОЙ АВИАЦИИ

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МАТЕМАТИЧЕСКИЙ АНАЛИЗ

ПОСОБИЕ

по выполнению практических заданий

*для студентов II курса
специальности 10.05.02
очной формы обучения*

Москва-2015

ФЕДЕРАЛЬНОЕ АГЕНТСТВО ВОЗДУШНОГО ТРАНСПОРТА
ФЕДЕРАЛЬНОЕ ГОСУДАРСТВЕННОЕ
БЮДЖЕТНОЕ ОБРАЗОВАТЕЛЬНОЕ УЧРЕЖДЕНИЕ
ВЫСШЕГО ПРОФЕССИОНАЛЬНОГО ОБРАЗОВАНИЯ
МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ ТЕХНИЧЕСКИЙ
УНИВЕРСИТЕТ ГРАЖДАНСКОЙ АВИАЦИИ (МГТУ ГА)

Кафедра высшей математики
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В пособии содержатся варианты контрольных домашних заданий и справочные материалы.

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ТРЕТИЙ СЕМЕСТР

КОНТРОЛЬНОЕ ДОМАШНЕЕ ЗАДАНИЕ №1

Двойные интегралы. Дифференциальные уравнения

Задание 1. Поменять порядок интегрирования.

Задания 2 – 3. Вычислить двойные интегралы.

Задание 4. Найти площадь фигуры, ограниченной данными линиями.

Задание 5. Пластинка D задана ограничивающими её кривыми, μ — поверхностная плотность. Найти массу пластинки.

Задания 6 – 7. Вычислить криволинейные интегралы вдоль линии L от точки M до точки N .

Задание 8. Для данного дифференциального уравнения методом изоклин построить интегральную кривую, проходящую через точку M .

Задания 9 – 14. Решить дифференциальные уравнения.

Вариант 1.

$$1. \int_{-2}^{-1} dy \int_0^{\sqrt{2+y}} f(x, y) dx + \int_{-1}^0 dy \int_0^{\sqrt{-y}} f(x, y) dx$$

$$2. \iint_D (12x^2y^2 + 16x^3y^3) dx dy; \quad D: x = 1, y = x^2, y = -\sqrt{x}$$

$$3. \iint_D ye^{\frac{xy}{2}} dx dy; \quad D: y = \ln 2, y = \ln 3, x = 2, x = 4$$

$$4. y^2 - 2y + x^2 = 0, y^2 - 4y + x^2 = 0, y = \frac{x}{\sqrt{3}}, y = \sqrt{3}x$$

$$5. D: x^2 + y^2 = 4, x^2 + y^2 = 9, y = -x, y = 0 (x \leq 0, y \geq 0), \mu = \frac{y - 4x}{x^2 + y^2}$$

$$6. \int_L (x + y) dx - (x - y) dy, L: \text{отрезок } MN, M(2; 0), N(4; 5)$$

$$7. \int_L (x^2y - 3x) dx + (y^2x + 2y) dy,$$

$$L: x = 3 \cos t, y = 3 \sin t, (y \geq 0), M(3; 0), N(-3; 0)$$

$$8. y' = y - x^2, M(1; 2)$$

$$9. 6x dx - 2y dy = 2y x^2 dy - 3xy^2 dx$$

10. $y' + y \operatorname{tg} x = \cos^2 x$, $y(\pi) = 0$ 11. $(1 + x^2) y'' + 2x y' = 2x$
 12. $y''' + 8y'' + 15y' = 0$ 13. $y'' - 2y' + y = 9e^{-2x}$
 14. $y'' + y' = 16x + 10$, $y(0) = 0$, $y'(0) = 0$

Вариант 2.

1. $\int_0^1 dy \int_{2y^2}^{3-y} f(x, y) dx$
 2. $\iint_D (9x^2y^2 + 48x^3y^3) dx dy$; $D: x = 1, y = \sqrt{x}, y = -x^2$
 3. $\iint_D y^2 \sin \frac{xy}{2} dx dy$; $D: x = 0, y = \sqrt{\pi}, y = \frac{x}{2}$
 4. $x^2 + 4x + y^2 = 0$, $x^2 + 8x + y^2 = 0$, $y = 0$, $y = -x$
 5. $D: x^2 + y^2 = 1$, $x^2 + y^2 = 4$, $y = \frac{x}{\sqrt{3}}$, $y = 0$ ($x \geq 0$, $y \geq 0$), $\mu = \frac{x + y}{x^2 + y^2}$
 6. $\int_L (x^2 - 2xy) dx + (y^2 - 2xy) dy$, $L: y = x^2$, $M(-1; 1)$, $N(1; 1)$
 7. $\int_L (-y dx + x dy)$,
 $L: x = 2 \cos t$, $y = 2 \sin t$, ($x \geq 0$, $y \geq 0$), $M(2; 0)$, $N(0; 2)$
 8. $y' = 2 + y^2$, $M(1; 2)$ 9. $y' \sin x = y \ln y$
 10. $y' + y \operatorname{ctg} x = \cos x$, $y\left(\frac{\pi}{2}\right) = \frac{1}{2}$ 11. $2xy'y'' = (y')^2 - 1$
 12. $y''' + 25y' = 0$ 13. $y'' + 2y' + y = 3x + 5$
 14. $y'' - 3y' + 2y = 10 \sin x$, $y(0) = 1$, $y'(0) = 0$

Вариант 3.

1. $\int_0^1 dy \int_0^{\sqrt{y}} f(x, y) dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f(x, y) dx$
 2. $\iint_D (36x^2y^2 - 96x^3y^3) dx dy$; $D: x = 1, y = \sqrt[3]{x}, y = -x^3$
 3. $\iint_D y \cos xy dx dy$; $D: y = \frac{\pi}{2}, y = \pi, x = 1, x = 2$

4. $y^2 + 6y + x^2 = 0$, $y^2 + 8y + x^2 = 0$, $x = 0$, $y = -\sqrt{3}x$
5. D : $x^2 + y^2 = 9$, $x^2 + y^2 = 25$, $y = -x$, $y = 0$ ($x \leq 0$, $y \geq 0$), $\mu = \frac{y - 2x}{x^2 + y^2}$
6. $\int_L y dx + \frac{x}{y} dy$, L : $y = e^{-x}$, $M(-1; e)$, $N(0; 1)$
7. $\int_L (x + 2y) dx + (x - y) dy$,
 L : $x = 4 \cos t$, $y = 4 \sin t$, ($x \geq 0$, $y \geq 0$), $M(4; 0)$, $N(0; 4)$
8. $y' = (y - 1)x$, $M(1; 3/2)$
9. $y' \sin x - y \cos x = 0$
10. $y' - 4xy = 2x e^{x^2}$, $y(0) = 1$
11. $y'' = \frac{y'}{x} + 1$
12. $y'''' - 7y'' = 0$
13. $y'' - 9y = 5x e^{2x}$
14. $y'' + 4y' + 5y = 25x$, $y(0) = 2$, $y'(0) = 0$

Вариант 4.

1. $\int_0^{3/2} dy \int_{2y^2}^{y+3} f(x, y) dx$
2. $\iint_D (18x^2y^2 + 32x^3y^3) dx dy$; D : $x = 1$, $y = x^3$, $y = -\sqrt[3]{x}$
3. $\iint_D y^2 e^{-\frac{xy}{4}} dx dy$; D : $x = 0$, $y = 2$, $y = x$
4. $x^2 - 2x + y^2 = 0$, $x^2 - 4x + y^2 = 0$, $y = 0$, $y = x$
5. D : $x^2 + y^2 = 9$, $x^2 + y^2 = 16$, $y = \sqrt{3}x$, $y = 0$ ($x \geq 0$, $y \geq 0$), $\mu = \frac{2x + 5y}{x^2 + y^2}$
6. $\int_L \frac{y^2 + 1}{y} dx - \frac{x}{y^2} dy$, L : отрезок MN , $M(1; 2)$, $N(2; 4)$
7. $\int_L (x^2 - y) dx + (x - y^2) dy$,
 L : $x = 5 \cos t$, $y = 5 \sin t$, ($x \leq 0$, $y \leq 0$), $M(-5; 0)$, $N(0; -5)$
8. $y' = 3 + y^2$, $M(1; 2)$
9. $(5 + y^2) + y' y (1 - x^2) = 0$
10. $y' - 4xy = 4x^3 e^{2x^2}$, $y(0) = 0$
11. $y'' \operatorname{ctg} x + 2y' = 0$
12. $y''' - 3y'' - 4y' = 0$
13. $y'' + 2y' + 5y = 17 \sin 2x$

14. $y'' - 6y' + 9y = 9x^2 - 3x - 4$, $y(0) = 1$, $y'(0) = 5$

Вариант 5.

1. $\int_0^1 dy \int_0^{\sqrt{y}} f(x, y) dx + \int_1^e dy \int_{\ln y}^1 f(x, y) dx$

2. $\iint_D (27x^2y^2 + 48x^3y^3) dx dy$; $D: x = 1, y = x^2, y = -\sqrt[3]{x}$

3. $\iint_D y \sin xy dx dy$; $D: y = \frac{\pi}{2}, y = \pi, x = 1, x = 2$

4. $y^2 - 8y + x^2 = 0, y^2 - 10y + x^2 = 0, y = \frac{x}{\sqrt{3}}, y = \sqrt{3}x$

5. $D: x^2 + y^2 = 4, x^2 + y^2 = 36, x = 0, y = x (x \leq 0, y \leq 0), \mu = \frac{-x - y}{x^2 + y^2}$

6. $\int_L (xy - x^2) dx + x dy$, $L: y = 2x^2, M(0; 0), N(1; 2)$

7. $\int_L (x + y) dx + 2x dy$,

$L: x = 2 \cos t, y = 2 \sin t, (x \geq 0), M(0; -2), N(0; 2)$

8. $y'(x^2 + 2) = y, M(2; 2)$

9. $y \ln y + x y' = 0$

10. $y' - 3x^2 y = x^2 e^{x^3}, y(0) = 0$

11. $x y'' - 2y' = -\frac{2}{x^2}$

12. $y''' - 3y'' - 4y' = 0$

13. $y'' - 2y' + y = (2x + 5) e^{2x}$

14. $y'' - 4y' + 13y = 26x + 5, y(0) = 1, y'(0) = 1$

Вариант 6.

1. $\int_0^4 dy \int_{3y/4}^{\sqrt{25-y^2}} f(x, y) dx$

2. $\iint_D (18x^2y^2 + 32x^3y^3) dx dy$; $D: x = 1, y = \sqrt[3]{x}, y = -x^2$

3. $\iint_D y^2 \cos \frac{xy}{2} dx dy$; $D: x = 0, y = \sqrt{\frac{\pi}{2}}, y = \frac{x}{2}$

4. $x^2 - 4x + y^2 = 0, x^2 - 8x + y^2 = 0, y = 0, y = \sqrt{3}x$

5. $D: x^2 + y^2 = 1, x^2 + y^2 = 16, x = 0, y = \frac{x}{\sqrt{3}} (x \geq 0, y \geq 0), \mu = \frac{x + 2y}{x^2 + y^2}$

6. $\int_L \frac{y}{x} dx + x dy, L: y = \ln x, M(1; 0), N(e; 1)$

7. $\int_L (2xy - y) dx + (x^2 + x) dy,$

$L: x = 3 \cos t, y = 3 \sin t, (y \leq 0), M(-3; 0), N(0; -3)$

8. $y' = y - x, M(9/2; 1)$

9. $(1 - x^2) y' + xy^2 + x = 0$

10. $y' - \frac{y}{x} = -\frac{2}{x^2}, y(1) = 1$

11. $xy'' + 2y' = 0$

12. $y''' + 5y'' - 14y' = 0$

13. $y'' + y = x^2 + 6$

14. $y'' - 5y' - 6y = e^x (-10x - 3), y(0) = 0, y'(0) = 8$

Вариант 7.

1. $\int_0^1 dx \int_{1-x^2}^1 f(x, y) dy + \int_1^e dx \int_{\ln x}^1 f(x, y) dy$

2. $\iint_D (18x^2y^2 + 32x^3y^3) dx dy; D: x = 1, y = x^3, y = -\sqrt{x}$

3. $\iint_D 4ye^{2xy} dx dy; D: y = \ln 3, y = \ln 4, x = \frac{1}{2}, x = 1$

4. $y^2 + 4y + x^2 = 0, y^2 + 6y + x^2 = 0, x = 0, y = -\frac{x}{\sqrt{3}}$

5. $D: x^2 + y^2 = 25, x^2 + y^2 = 36, y = -x, y = 0 (x \geq 0, y \leq 0), \mu = \frac{x - y}{x^2 + y^2}$

6. $\int_L (x^2 + y) dx - (y^2 + x) dy, L: \text{отрезок } MN, M(1; 2), N(3; 5)$

7. $\int_L xy dx + 2y dy,$

$L: x = \cos t, y = \sin t, (x \leq 0), M(0; 1), N(0; -1)$

8. $y' = xy, M(0; -1)$

9. $\sqrt{4 - x^2} y' + x (y^2 + 1) = 0$

10. $y' - \frac{2y}{x+1} = (x+1)^3, y(0) = \frac{1}{2}$

11. $(1 + \sin x) y'' = y' \cos x$

12. $y'''' - 81y = 0$

13. $y'' - 4y' + 3y = -4x e^x$

14. $y'' + 6y' + 9y = 25 e^{2x}, y(0) = 3, y'(0) = 2$

Вариант 8.

$$1. \int_0^1 dy \int_0^{y^2+1} f(x, y) dx$$

$$2. \iint_D (27x^2y^2 + 48x^3y^3) dx dy; \quad D: x = 1, y = \sqrt{x}, y = -x^3$$

$$3. \iint_D 4y^2 \sin xy dx dy; \quad D: x = 0, y = \sqrt{\frac{\pi}{2}}, y = x$$

$$4. x^2 + 2x + y^2 = 0, x^2 + 10x + y^2 = 0, y = 0, y = \sqrt{3}x$$

$$5. D: x^2 + y^2 = 4, x^2 + y^2 = 25, x = 0, y = \frac{x}{\sqrt{3}} (x \leq 0, y \leq 0), \mu = \frac{-2x - 3y}{x^2 + y^2}$$

$$6. \int_L (xy - x) dx + \frac{x^2}{2} dy, \quad L: y = 2\sqrt{x}, M(0; 0), N(1; 2)$$

$$7. \int_L (x^2 + y^2) dx + (x^2 - y^2) dy,$$

$$L: x = 6 \cos t, y = 6 \sin t, (y \geq 0), M(6; 0), N(-6; 0)$$

$$8. yy' = -\frac{x}{2}, M(4; 2)$$

$$9. y' y \sqrt{\frac{1-x^2}{1-y^2}} + 1 = 0$$

$$10. y' + \frac{y}{2x} = x, y(1) = 0$$

$$11. x^3 y'' + x^2 y' = 1$$

$$12. y''' - 9y'' + 8y' = 0$$

$$13. y'' - y' - 2y = (1 - 2x) e^x$$

$$14. y'' - 2y' + y = 16e^x, y(0) = 1, y'(0) = 2$$

Вариант 9.

$$1. \int_0^1 dy \int_0^{\sqrt[3]{y}} f(x, y) dx + \int_1^2 dy \int_0^{2-y} f(x, y) dx$$

$$2. \iint_D (4xy + 3x^2y^2) dx dy; \quad D: x = 1, y = x^2, y = -\sqrt{x}$$

$$3. \iint_D y \cos 2xy dx dy; \quad D: y = \frac{\pi}{2}, y = \pi, x = \frac{1}{2}, x = 1$$

$$4. y^2 - 6y + x^2 = 0, y^2 - 10y + x^2 = 0, y = x, x = 0$$

$$5. D: x^2 + y^2 = 9, x^2 + y^2 = 36, y = -\sqrt{3}x, y = 0 (x \leq 0, y \geq 0), \mu = \frac{2y - 4x}{x^2 + y^2}$$

$$6. \int_L \frac{y}{x^2 + y^2} dx + \frac{x}{x^2 + y^2} dy, \quad L: \text{отрезок } MN, \quad M(1; 2), \quad N(3; 6)$$

$$7. \int_L (x + y\sqrt{x^2 + y^2}) dx + x dy,$$

$$L: x = \cos t, \quad y = \sin t, \quad (y \leq 0), \quad M(-1; 0), \quad N(0; -1)$$

$$8. y' = x + 2y, \quad M(3; 0)$$

$$9. \sqrt{4 + x^2} dx - 4y dy = x^2 y dy$$

$$10. y' - \frac{y}{x} = x^2, \quad y(1) = 0$$

$$11. x^5 y'' + x^4 y' = 9$$

$$12. y''' - 6y'' + 9y' = 0$$

$$13. y'' + 6y' + 13y = 75 \cos 2x$$

$$14. y'' + y = 4e^x, \quad y(0) = 4, \quad y'(0) = -3$$

Вариант 10.

$$1. \int_0^4 dx \int_{\sqrt{x}}^{2\sqrt{x}} f(x, y) dy$$

$$2. \iint_D (12xy + 9x^2y^2) dx dy; \quad D: x = 1, \quad y = \sqrt{x}, \quad y = -x^2$$

$$3. \iint_D y^2 e^{-\frac{xy}{8}} dx dy; \quad D: x = 0, \quad y = 2, \quad y = \frac{x}{2}$$

$$4. x^2 - 2x + y^2 = 0, \quad x^2 - 4x + y^2 = 0, \quad y = \frac{x}{\sqrt{3}}, \quad y = \sqrt{3}x$$

$$5. D: x^2 + y^2 = 1, \quad x^2 + y^2 = 9, \quad y = -\sqrt{3}x, \quad y = 0 \quad (x \geq 0, \quad y \leq 0), \quad \mu = \frac{x - y}{x^2 + y^2}$$

$$6. \int_L (x^2 - 2y) dx + (y^2 - 2x) dy, \quad L: \text{отрезок } MN, \quad M(-4; 0), \quad N(0; 2)$$

$$7. \int_L x^2 y dx - xy^2 dy,$$

$$L: x = 2 \cos t, \quad y = 2 \sin t, \quad (x \leq 0, \quad y \geq 0), \quad M(0; 2), \quad N(-2; 0)$$

$$8. 3yy' = x, \quad M(-3; -2)$$

$$9. x \sqrt{1 + y^2} + y y' (1 + x^2) = 0$$

$$10. y' - \frac{y}{x} = x \sin x, \quad y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

$$11. x^2 y'' + x y' = 1$$

$$12. y''' - 2y'' - 8y' = 0$$

$$13. y'' + 2y' + y = 2 - 3x^2$$

$$14. y'' + 81y = 162e^{9x}, \quad y(0) = 0, \quad y'(0) = 9$$

Вариант 11.

$$1. \int_0^1 dy \int_{-\sqrt{y}}^0 f(x, y) dx + \int_1^2 dy \int_{-\sqrt{2-y}}^0 f(x, y) dx$$

$$2. \iint_D (8xy + 9x^2y^2) dx dy; \quad D: x = 1, y = \sqrt[3]{x}, y = -x^3$$

$$3. \iint_D 12y \sin 2xy dx dy; \quad D: y = \frac{\pi}{4}, y = \frac{\pi}{2}, x = 2, x = 3$$

$$4. y^2 + 2y + x^2 = 0, y^2 + 4y + x^2 = 0, x = 0, y = x$$

$$5. D: x^2 + y^2 = 1, x^2 + y^2 = 36, x = 0, y = -x (x \geq 0, y \leq 0), \mu = \frac{2x - y}{x^2 + y^2}$$

$$6. \int_L \frac{y}{x} dx + (x^3 + 1) dy, \quad L: y = \ln x, M(1; 0), N(e; 1)$$

$$7. \int_L (x^2 + \sqrt{x^2 + y^2}) dx + (y - \sqrt{x^2 + y^2}) dy,$$

$$L: x = 4 \cos t, y = 4 \sin t, (x \geq 0, y \leq 0), M(0; -4), N(4; 0)$$

$$8. x^2 - y^2 + 2xyy' = 0, M(-2; 1) \quad 9. y(1 - \ln y) + xy' = 0$$

$$10. xy' + y = \ln x, y(1) = 1 \quad 11. 2xy'' = y'$$

$$12. y''' - 6y'' + 12y' - 8y = 0 \quad 13. y'' + y' - 6y = 10e^{2x}$$

$$14. y'' + y = 1, y\left(\frac{\pi}{2}\right) = 0, y'\left(\frac{\pi}{2}\right) = 0$$

Вариант 12.

$$1. \int_0^1 dx \int_{2x+1}^{4-x^2} f(x, y) dy$$

$$2. \iint_D (24xy + 18x^2y^2) dx dy; \quad D: x = 1, y = x^3, y = -\sqrt[3]{x}$$

$$3. \iint_D y^2 \cos xy dx dy; \quad D: x = 0, y = \sqrt{\pi}, y = x$$

$$4. x^2 + 2x + y^2 = 0, x^2 + 6x + y^2 = 0, y = 0, y = x$$

$$5. D: x^2 + y^2 = 9, x^2 + y^2 = 25, x = 0, y = -\frac{x}{\sqrt{3}} (x \leq 0, y \geq 0), \mu = \frac{2y - x}{x^2 + y^2}$$

$$6. \int_L (2xy + y^2) dx - x dy, \quad L: y = 2x^2, \quad M(-1; 2), \quad N(0; 0)$$

$$7. \int_L y^2 dx - x^2 dy,$$

$$L: x = 5 \cos t, \quad y = 5 \sin t, \quad (x \leq 0, \quad y \leq 0), \quad M(-5; 0), \quad N(0; -5)$$

$$8. y' = y - x, \quad M(2; 1)$$

$$9. 2x + 2xy^2 + (2 - x^2) y' = 0$$

$$10. y' + \frac{y}{x} = 3x, \quad y(1) = 1$$

$$11. x y'' + y' = x + 1$$

$$12. y''' + 2y'' - 24y' = 0$$

$$13. y'' + 3y' + 2y = (6x - 1) e^x$$

$$14. y'' + 9y = 18x + 9, \quad y(0) = 0, \quad y'(0) = 5$$

Вариант 13.

$$1. \int_0^1 dy \int_{-y}^0 f(x, y) dx + \int_1^{\sqrt{2}} dy \int_{-\sqrt{2-y^2}}^0 f(x, y) dx$$

$$2. \iint_D (12xy + 27x^2y^2) dx dy; \quad D: x = 1, \quad y = x^2, \quad y = -\sqrt[3]{x}$$

$$3. \iint_D ye^{\frac{xy}{4}} dx dy; \quad D: y = \ln 2, \quad y = \ln 3, \quad x = 4, \quad x = 8$$

$$4. y^2 - 4y + x^2 = 0, \quad y^2 - 6y + x^2 = 0, \quad y = \sqrt{3}x, \quad x = 0$$

$$5. D: x^2 + y^2 = 16, \quad x^2 + y^2 = 36, \quad y = x, \quad y = 0 \quad (x \geq 0, \quad y \geq 0), \quad \mu = \frac{x + 6y}{x^2 + y^2}$$

$$6. \int_L 2xy dx - x^2 dy, \quad L: y = 2x^2, \quad M(0; 0), \quad N(1; 2)$$

$$7. \int_L (-y dx + (2xy + x) dy),$$

$$L: x = 3 \cos t, \quad y = 3 \sin t, \quad (y \geq 0), \quad M(3; 0), \quad N(-3; 0)$$

$$8. y' = x^2 - y, \quad M(0; 1)$$

$$9. 2x dx - y dy = y x^2 dy - x y^2 dx$$

$$10. y' + \frac{3y}{x} = \frac{2}{x^3}, \quad y(1) = 1$$

$$11. y'' \operatorname{tg} x = y' + 1$$

$$12. y''' + 4y'' + 4y' = 0$$

$$13. y'' + 2y' - 3y = 30 \cos 3x$$

$$14. y'' - 2y' = 2e^x, \quad y(0) = 0, \quad y'(0) = 0$$

Вариант 14.

$$1. \int_0^2 dx \int_{x^2/4}^{2\sqrt{x}} f(x, y) dy$$

$$2. \iint_D (8xy + 18x^2y^2) dx dy; \quad D: x = 1, y = \sqrt[3]{x}, y = -x^2$$

$$3. \iint_D 4y^2 \sin 2xy dx dy; \quad D: x = 0, y = \sqrt{2\pi}, y = 2x$$

$$4. x^2 - 2x + y^2 = 0, x^2 - 8x + y^2 = 0, y = \frac{x}{\sqrt{3}}, y = \sqrt{3}x$$

$$5. D: x^2 + y^2 = 4, x^2 + y^2 = 16, x = 0, y = \sqrt{3}x (x \leq 0, y \leq 0), \mu = \frac{-2y - 3x}{x^2 + y^2}$$

$$6. \int_L (x + y)^2 dx - (x^2 + y^2) dy, L: \text{отрезок } MN, M(0; 1), N(1; 0)$$

$$7. \int_L (x - y) dx + dy,$$

$$L: x = 2 \cos t, y = 2 \sin t, (y \leq 0), M(-2; 0), N(2; 0)$$

$$8. yy' = -2x, M(0; 5)$$

$$9. (1 + e^x) y y' = e^x$$

$$10. y' + \frac{y}{x} = e^x, y(1) = 0$$

$$11. x y'' + y' + x = 0$$

$$12. y''' + 3y'' - 4y' = 0$$

$$13. y'' - 3y' + 2y = -5e^x$$

$$14. y'' + y = -\sin(2x), y(\pi) = 1, y'(\pi) = 1$$

Вариант 15.

$$1. \int_0^1 dy \int_0^{y^3} f(x, y) dx + \int_1^2 dy \int_0^{2-y} f(x, y) dx$$

$$2. \iint_D (24xy - 48x^3y^3) dx dy; \quad D: x = 1, y = x^2, y = -\sqrt{x}$$

$$3. \iint_D 2y \cos 2xy dx dy; \quad D: y = \frac{\pi}{4}, y = \frac{\pi}{2}, x = 1, x = 2$$

$$4. y^2 + 2y + x^2 = 0, y^2 + 6y + x^2 = 0, x = 0, y = \frac{x}{\sqrt{3}}$$

$$5. D: x^2 + y^2 = 25, x^2 + y^2 = 49, y = 0, y = -\sqrt{3}x (x \leq 0, y \geq 0), \mu = \frac{4y - x}{x^2 + y^2}$$

$$6. \int_L y^2 dx + y dy, \quad L: y = \sin x, \quad M(-\pi; 0), \quad N(0; 0)$$

$$7. \int_L y dx - x dy,$$

$$L: x = \sqrt{2} \cos t, \quad y = \sqrt{2} \sin t, \quad (x \geq 0), \quad M(0; -\sqrt{2}), \quad N(0; \sqrt{2})$$

$$8. y' = \frac{2x}{3y}, \quad M(1; 1)$$

$$9. \sqrt{5 + y^2} dx + 4(x^2 y + y) dy = 0$$

$$10. y' + \frac{2xy}{1 + x^2} = \frac{3x^2}{1 + x^2}, \quad y(0) = 0$$

$$11. y'' \operatorname{tg} x = y'$$

$$12. y''' - 9y'' + 8y' = 0$$

$$13. y'' + y' - 2y = 9e^x$$

$$14. y'' + y = 48 \cos 5x + 72 \sin 5x, \quad y(0) = 0, \quad y'(0) = 0$$

Вариант 16.

$$1. \int_0^4 dx \int_{x/2+1}^{7-x} f(x, y) dy$$

$$2. \iint_D (6xy + 24x^3y^3) dx dy; \quad D: x = 1, \quad y = \sqrt{x}, \quad y = -x^2$$

$$3. \iint_D y^2 e^{-\frac{xy}{2}} dx dy; \quad D: x = 0, \quad y = \sqrt{2}, \quad y = x$$

$$4. x^2 + 2x + y^2 = 0, \quad x^2 + 4x + y^2 = 0, \quad y = 0, \quad y = -x$$

$$5. D: x^2 + y^2 = 9, \quad x^2 + y^2 = 16, \quad x = 0, \quad y = -\sqrt{3}x \quad (x \leq 0, y \geq 0), \quad \mu = \frac{2y - 5x}{x^2 + y^2}$$

$$6. \int_L 2y dx + (3x - y) dy, \quad L: y = \sqrt{x}, \quad M(1; 1), \quad N(4; 2)$$

$$7. \int_L (-x dx + y dy),$$

$$L: x = 3 \cos t, \quad y = 3 \sin t, \quad (x \geq 0, y \geq 0), \quad M(3; 0), \quad N(0; 3)$$

$$8. yy' + x = 0, \quad M(-2; -3)$$

$$9. (e^{2x} + 2) dy + y e^{2x} dx = 0$$

$$10. y' + \frac{y}{x} = \sin x, \quad y(\pi) = 1$$

$$11. x y'' - y' + \frac{1}{x} = 0$$

$$12. y''' + 36y' = 0$$

$$13. y'' - 6y' + 9y = 4x e^x$$

$$14. y'' - 3y' + 2y = 24 e^{-2x}, \quad y(0) = -1, \quad y'(0) = 4$$

Вариант 17.

$$1. \int_{-2}^{-1} dy \int_0^{y+2} f(x, y) dx + \int_{-1}^0 dy \int_0^{\sqrt{-y}} f(x, y) dx$$

$$2. \iint_D (4xy + 16x^3y^3) dx dy; \quad D: x = 1, y = \sqrt[3]{x}, y = -x^3$$

$$3. \iint_D y \sin xy dx dy; \quad D: y = \pi, y = 2\pi, x = \frac{1}{2}, x = 1$$

$$4. y^2 - 2y + x^2 = 0, y^2 - 10y + x^2 = 0, y = \frac{x}{\sqrt{3}}, y = \sqrt{3}x$$

$$5. D: x^2 + y^2 = 4, x^2 + y^2 = 49, y = \sqrt{3}x, y = 0 \quad (x \leq 0, y \leq 0), \mu = \frac{-2x - 4y}{x^2 + y^2}$$

$$6. \int_L (2xy^2 - 1)y dx - (3xy^2 + 5)x dy, L: \text{отрезок } MN, M(0; 0), N(2; 4)$$

$$7. \int_L (x^2 - y) dx + (x + y^2) dy,$$

$$L: x = 2 \cos t, y = 2 \sin t, (y \geq 0), M(2; 0), N(-2; 0)$$

$$8. xy' = 2y, M(2; 3)$$

$$9. x dx - 3y dy = y x^2 dy - x y^2 dx$$

$$10. y' - \frac{y}{x+1} = e^x (x+1), y(0) = 1$$

$$11. y'' \operatorname{ctg} x = 2y'$$

$$12. y''' + 3y'' + 3y' + y = 0$$

$$13. y'' + 3y' + 2y = 12x^2 + 8x$$

$$14. y'' - 5y' + 4y = 3e^{4x}, y(0) = 0, y'(0) = 4$$

Вариант 18.

$$1. \int_0^3 dx \int_0^{\sqrt{4-x}} f(x, y) dy$$

$$2. \iint_D (4xy + 16x^3y^3) dx dy; \quad D: x = 1, y = x^3, y = -\sqrt[3]{x}$$

$$3. \iint_D y^2 \cos 2xy dx dy; \quad D: x = 0, y = \sqrt{\frac{\pi}{2}}, y = \frac{x}{2}$$

$$4. x^2 - 2x + y^2 = 0, x^2 - 6x + y^2 = 0, y = 0, y = \frac{x}{\sqrt{3}}$$

$$5. D: x^2 + y^2 = 1, x^2 + y^2 = 16, x = 0, y = \sqrt{3}x \quad (x \geq 0, y \geq 0), \mu = \frac{x + 3y}{x^2 + y^2}$$

$$6. \int_L (x^2 + 4xy) dx + (2xy + y^2) dy, \quad L: y = x^2, \quad M(1; 1), \quad N(2; 4)$$

$$7. \int_L (x + y) dx + (x - y) dy,$$

$$L: x = 4 \cos t, \quad y = 4 \sin t, \quad (x \leq 0, \quad y \leq 0), \quad M(-4; 0), \quad N(0; -4)$$

$$8. 2(y + y') = x + 3, \quad M(1; 1/2)$$

$$9. (x^2 y + 9y) dy + \sqrt{2 + y^2} dx = 0$$

$$10. x y' + y = x^5, \quad y(1) = 0$$

$$11. (1 + x^2) y'' + 2x y' = 2$$

$$12. y''' + 4y'' - 5y' = 0$$

$$13. y'' + 25y = 50 e^{5x}$$

$$14. y'' - y = 2x, \quad y(0) = 0, \quad y'(0) = 0$$

Вариант 19.

$$1. \int_0^1 dy \int_0^y f(x, y) dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f(x, y) dx$$

$$2. \iint_D (44xy + 16x^3 y^3) dx dy; \quad D: x = 1, \quad y = x^2, \quad y = -\sqrt[3]{x}$$

$$3. \iint_D 8ye^{4xy} dx dy; \quad D: y = \ln 3, \quad y = \ln 4, \quad x = \frac{1}{4}, \quad x = \frac{1}{2}$$

$$4. y^2 + 4y + x^2 = 0, \quad y^2 + 10y + x^2 = 0, \quad x = 0, \quad y = -x$$

$$5. D: x^2 + y^2 = 9, \quad x^2 + y^2 = 49, \quad y = 0, \quad y = -\sqrt{3}x \quad (x \leq 0, \quad y \geq 0), \quad \mu = \frac{3y - x}{x^2 + y^2}$$

$$6. \int_L \frac{y^2}{x} dx - x^2 dy, \quad L: y = \ln x, \quad M(1; 0), \quad N(e; 1)$$

$$7. \int_L (2x - y) dx + x dy,$$

$$L: x = 3 \cos t, \quad y = 3 \sin t, \quad (y \leq 0), \quad M(-3; 0), \quad N(3; 0)$$

$$8. yy' = -x, \quad M(2; 3)$$

$$9. x\sqrt{5+y^2} dx + y\sqrt{4+x^2} dy = 0$$

$$10. y' - y \operatorname{ctg} x = 2x \sin x, \quad y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4}$$

$$11. x y'' + y' = \frac{1}{\sqrt{x}}$$

$$12. y''' + y'' - 2y' = 0$$

$$13. y'' - 3y' + 2y = -5e^x$$

$$14. y'' - 64y = 128 \cos 8x, \quad y(0) = 0, \quad y'(0) = 0$$

Вариант 20.

$$1. \int_0^1 dx \int_{-\sqrt{1-x^2}}^{1-x} f(x, y) dy$$

$$2. \iint_D (4xy + 176x^3y^3) dx dy; \quad D: x = 1, y = \sqrt[3]{x}, y = -x^3$$

$$3. \iint_D 3y^2 \sin \frac{xy}{2} dx dy; \quad D: x = 0, y = \sqrt{\frac{4\pi}{3}}, y = \frac{2}{3}x$$

$$4. x^2 + 2x + y^2 = 0, x^2 + 6x + y^2 = 0, y = 0, y = x$$

$$5. D: x^2 + y^2 = 1, x^2 + y^2 = 4, x = 0, y = \frac{x}{\sqrt{3}} (x \geq 0, y \geq 0), \mu = \frac{x + 2y}{x^2 + y^2}$$

$$6. \int_L \left(y - \frac{1}{y} \right) dx + \left(\frac{x}{y} - 2 \right) dy, L: y = \frac{1}{x}, M(1; 1), N \left(2; \frac{1}{2} \right)$$

$$7. \int_L (x + y) dx + (2x - y) dy,$$

$$L: x = 5 \cos t, y = 5 \sin t, (x \geq 0, y \leq 0), M(0; -5), N(5; 0)$$

$$8. 3yy' = x, M(1; 1)$$

$$9. 6x dx - y dy = y x^2 dy - 3x y^2 dx$$

$$10. y' - \frac{y}{x} = x^2, y(1) = 0$$

$$11. x^4 y'' + x^3 y' = 1$$

$$12. y''' + 6y'' + 5y' = 0$$

$$13. y'' + y = 2 \cos 7x - 3 \sin 7x$$

$$14. y'' + 3y' + 2y = 1 - 2x^2, y(0) = 0, y'(0) = 0$$

Вариант 21.

$$1. \int_0^1 dx \int_0^{x^2} f(x, y) dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f(x, y) dy$$

$$2. \iint_D (xy - 4x^3y^3) dx dy; \quad D: x = 1, y = x^3, y = -\sqrt{x}$$

$$3. \iint_D y \cos xy dx dy; \quad D: y = \pi, y = 3\pi, x = \frac{1}{2}, x = 1$$

$$4. y^2 - 2y + x^2 = 0, y^2 - 4y + x^2 = 0, y = \frac{x}{\sqrt{3}}, x = 0$$

$$5. D: x^2 + y^2 = 36, x^2 + y^2 = 49, y = 0, y = -x (x \leq 0, y \geq 0), \mu = \frac{4y - 2x}{x^2 + y^2}$$

$$6. \int_L (x^2 + y^2) dx + \frac{x^3}{y} dy, \quad L: y = e^{2x}, \quad M(0; 1), \quad N(1; e^2)$$

$$7. \int_L (x + y) dx + (x - y) dy,$$

$$L: x = 3 \cos t, \quad y = 3 \sin t, \quad (y \leq 0), \quad M(-3; 0), \quad N(3; 0)$$

$$8. y' = x + 2y, \quad M(1; 2)$$

$$9. (2 - e^x) dy + 3e^x \operatorname{tg} y dx = 0$$

$$10. y' - y \operatorname{tg} x = 1, \quad y(0) = 0$$

$$11. y'' x \ln x = y'$$

$$12. y''' + 6y'' + 9y' = 0$$

$$13. y'' + y = 16 \cos 3x - 24 \sin 3x$$

$$14. y'' + 6y' + 5y = 84 e^{2x}, \quad y(0) = -1, \quad y'(0) = 1$$

Вариант 22.

$$1. \int_0^{1/4} dy \int_y^{\sqrt{y}} f(x, y) dx$$

$$2. \iint_D (4xy + 176x^3y^3) dx dy; \quad D: x = 1, \quad y = \sqrt{x}, \quad y = -x^3$$

$$3. \iint_D y^2 e^{-\frac{xy}{2}} dx dy; \quad D: x = 0, \quad y = 1, \quad y = \frac{x}{2}$$

$$4. x^2 - 2x + y^2 = 0, \quad x^2 - 4x + y^2 = 0, \quad y = 0, \quad y = \sqrt{3}x$$

$$5. D: x^2 + y^2 = 1, \quad x^2 + y^2 = 9, \quad y = \frac{x}{\sqrt{3}}, \quad y = 0 \quad (x \leq 0, \quad y \leq 0), \quad \mu = \frac{-2x - y}{x^2 + y^2}$$

$$6. \int_L \left(y + \frac{1}{y} \right) dx - \frac{x}{y^2} dy, \quad L: y = x^3, \quad M(1; 1), \quad N(2; 8)$$

$$7. \int_L y^2 dx + xy dy,$$

$$L: x = 3 \cos t, \quad y = 3 \sin t, \quad (x \leq 0), \quad M(0; 3), \quad N(0; -3)$$

$$8. x^2 - y^2 + 2xyy' = 0, \quad M(2; 1)$$

$$9. y' = (2y + 1) \operatorname{ctg} x$$

$$10. y' - \frac{3y}{x} = x, \quad y(1) = 6$$

$$11. y'' - \frac{y'}{x(2 + \ln x)} = 2 + \ln x$$

$$12. y''' + y' = 0$$

$$13. y'' + 4y' + 4y = 8x^2 + 6$$

$$14. y'' - y' = 2x, \quad y(0) = 0, \quad y'(0) = 0$$

Вариант 23.

$$1. \int_0^2 dy \int_{y/2}^y f(x, y) dx + \int_2^4 dy \int_{y/2}^2 f(x, y) dx$$

$$2. \iint_D (9x^2y^2 + 25x^4y^4) dx dy; \quad D: x = 1, y = \sqrt{x}, y = -x^2$$

$$3. \iint_D y \sin 2xy dx dy; \quad D: y = \frac{\pi}{2}, y = \frac{3\pi}{2}, x = \frac{1}{2}, x = 3$$

$$4. y^2 + 6y + x^2 = 0, y^2 + 8y + x^2 = 0, x = 0, y = x$$

$$5. D: x^2 + y^2 = 1, x^2 + y^2 = 49, y = -\sqrt{3}x, y = 0 (x \leq 0, y \geq 0), \mu = \frac{3y - 2x}{x^2 + y^2}$$

$$6. \int_L 2xy dx + (x^2 + 2) dy, \quad L: y = \frac{x^2}{4}, M(-2; 1), N(0; 0)$$

$$7. \int_L \left(-\frac{y}{x^2 + y^2} dx + \frac{x}{x^2 + y^2} dy \right),$$

$$L: x = 4 \cos t, y = 4 \sin t, (x \leq 0, y \geq 0), M(0; 4), N(-4; 0)$$

$$8. y' = x(y - 1), M(1; 1/2)$$

$$9. \sqrt{3 + y^2} + \sqrt{1 - x^2} y y' = 0$$

$$10. y' - \frac{y}{x} = \ln x, y(1) = 0$$

$$11. x^5 y'' + x^4 y' = 1$$

$$12. y''' - 3y'' + 2y' = 0$$

$$13. y'' + y' = 4x - 1$$

$$14. y'' - 2y' = e^x (3x - 1), y(0) = 2, y'(0) = 0$$

Вариант 24.

$$1. \int_0^1 dx \int_{x^2}^{2x^2+1} f(x, y) dy$$

$$2. \iint_D (54x^2y^2 + 150x^4y^4) dx dy; \quad D: x = 1, y = x^2, y = -\sqrt[3]{x}$$

$$3. \iint_D y^2 \cos xy dx dy; \quad D: x = 0, y = \sqrt{\pi}, y = 2x$$

$$4. x^2 + 4x + y^2 = 0, x^2 + 8x + y^2 = 0, y = 0, y = -x$$

$$5. D: x^2 + y^2 = 1, x^2 + y^2 = 25, x = 0, y = -\sqrt{3}x (x \geq 0, y \leq 0), \mu = \frac{x - 4y}{x^2 + y^2}$$

$$6. \int_L (3x^2y + 1) dx + (x^3 + 2) dy, \quad L: y = 2\sqrt{x}, \quad M(0;0), \quad N(1;2)$$

$$7. \int_L x^3 dx - y^3 dy,$$

$$L: x = 2 \cos t, \quad y = 2 \sin t, \quad (x \geq 0), \quad M(0; -2), \quad N(0; 2)$$

$$8. y' = y - x^2, \quad M(-3; 4)$$

$$9. x\sqrt{1-y^2} dx + y\sqrt{1-x^2} dy = 0$$

$$10. y' + y \cos x = \cos x e^{-\sin x}, \quad y(0) = 1$$

$$11. xy'' + y' = 3x + 2$$

$$12. y'''' - 16y = 0$$

$$13. y'' - 2y' + y = e^{6x}$$

$$14. y'' - 4y' + 3y = 10 \cos x, \quad y(0) = 1, \quad y'(0) = 2$$

Вариант 25.

$$1. \int_1^2 dy \int_{-\sqrt{y-1}}^{\sqrt{y-1}} f(x, y) dx + \int_2^5 dy \int_{-\sqrt{y-1}}^{3-y} f(x, y) dx$$

$$2. \iint_D (xy - 9x^5y^5) dx dy; \quad D: x = 1, \quad y = \sqrt[3]{x}, \quad y = -x^2$$

$$3. \iint_D 6ye^{\frac{xy}{3}} dx dy; \quad D: y = \ln 2, \quad y = \ln 3, \quad x = 3, \quad x = 6$$

$$4. y^2 - 4y + x^2 = 0, \quad y^2 - 8y + x^2 = 0, \quad x = 0, \quad y = \sqrt{3x}$$

$$5. D: x^2 + y^2 = 4, \quad x^2 + y^2 = 16, \quad y = 0, \quad y = -x \quad (x \geq 0, \quad y \leq 0), \quad \mu = \frac{3x - 4y}{x^2 + y^2}$$

$$6. \int_L (y^2 + x) dx + \frac{2x}{y} dy, \quad L: y = e^{3x}, \quad M(0;1), \quad N(1; e^3)$$

$$7. \int_L xy dx + y^2 dy,$$

$$L: x = 4 \cos t, \quad y = 4 \sin t, \quad (x \leq 0, \quad y \geq 0), \quad M(0;4), \quad N(-4;0)$$

$$8. xy' = 2y, \quad M(1;3)$$

$$9. y(5 + \ln y) + xy' = 0$$

$$10. y' - y \cos x = \cos^2 x e^{\sin x}, \quad y(0) = 0$$

$$11. x^4 y'' + x^3 y' = 4$$

$$12. y'''' - 9y'' = 0$$

$$13. y'' + y' = x$$

$$14. y'' + 4y = e^{-2x}, \quad y(0) = 0, \quad y'(0) = 0$$

КОНТРОЛЬНОЕ ДОМАШНЕЕ ЗАДАНИЕ №2

Теория функций комплексного переменного. Операционное исчисление

Задания 1 – 2. Представить числа в тригонометрической и показательной формах. Изобразить числа на комплексной плоскости.

Задание 3. Представить число в алгебраической, тригонометрической и показательной формах.

Задание 4. Найти особые точки функции. Определить их тип.

Задание 5. Найти вычеты функции в особых точках.

Задания 6 – 7. Вычислить интегралы с помощью вычетов.

Задание 8. Найти изображение по оригиналу.

Задания 9. Найти оригинал по изображению.

Задания 10. Решить задачу Коши операционным методом.

Задания 11. Решить систему дифференциальных уравнений операционным методом.

Вариант 1.

- | | | |
|--|--|--|
| 1. $z = -3i$ | 2. $z = -\sqrt{2} - \sqrt{2}i$ | 3. $z = \frac{2 + 3i}{7 - 5i}$ |
| 4. $f(z) = \frac{e^z - 1}{z^3(z + 1)^2}$ | 5. $f(z) = \frac{1}{z^4 - z^2}$ | 6. $\oint_{ z =1} z^2 \cdot \sin \frac{1}{z} dz$ |
| 7. $\oint_{ z-1/2 =1} \frac{e^z + 1}{z(z - 1)} dz$ | 8. $f(t) = 4t \sin t - e^{2t} \cos 4t$ | 11. $\begin{cases} x' = x - y, \\ y' = x + y, \end{cases}$ |
| 9. $F(p) = \frac{3p^2 - p + 2}{(p - 1)(p^2 + 4p + 5)}$ | | $x(0) = 1, y(0) = 0$ |
| 10. $x'' + x = 2 \cos t, x(0) = 0, x'(0) = 1$ | | |

Вариант 2.

1. $z = 7i$
2. $z = 4i + 4$
3. $z = \frac{-4 - 2i}{3 + 7i}$
4. $f(z) = \frac{e^{1/z}}{z^4 - 1}$
5. $f(z) = \frac{z^2}{(z^2 + 1)(z - 3)}$
6. $\oint_{|z-i|=1} \frac{1}{(z^2 + 1)^3} dz$
7. $\oint_{|z-1|=3} \frac{ze^z}{\sin z} dz$
8. $f(t) = 3t^2 - e^{-2t} \cos 5t$
9. $F(p) = \frac{4p + 5}{(p - 2)(p^2 + 4p + 15)}$
10. $x'' + x = 6e^{-t}$, $x(0) = 3$, $x'(0) = 1$
11. $\begin{cases} x' = x + 3y + 2, \\ y' = x - y + 1, \end{cases}$
 $x(0) = -1$, $y(0) = 2$

Вариант 3.

1. $z = -4$
2. $z = -6\sqrt{3}i - 6$
3. $z = \frac{3i - 5}{2i + 4}$
4. $f(z) = \frac{1}{e^z + 1}$
5. $f(z) = z^2 e^{1/z}$
6. $\oint_{|z|=4} \operatorname{ctg} z dz$
7. $\oint_{|z+1|=3} \frac{z^2 + \cos z}{z^3} dz$
8. $f(t) = 3e^{2t} \sin t - 2e^{-t} \cos 5t$
9. $F(p) = \frac{p + 3}{p^3 + 2p^2 + 3p}$
10. $x'' + x' = t^2 + 2t$, $x(0) = 0$, $x'(0) = -2$
11. $\begin{cases} x' = -x + 3y + 1, \\ y' = x + y, \end{cases}$
 $x(0) = 1$, $y(0) = 2$

Вариант 4.

1. $z = 2i$
2. $z = 6 + 2\sqrt{3}i$
3. $z = \frac{5i + 1}{7 - 6i}$
4. $f(z) = \operatorname{ctg} \pi z$
5. $f(z) = \frac{z^2}{(z - 2)^3}$
6. $\oint_{|z|=2} \frac{e^z}{z^3(z + 1)} dz$
7. $\oint_{|z-1|=2} \frac{z - \sin z}{2z^4} dz$
8. $f(t) = (3t^2 - 8t)e^{-t} - 4e^{15t} \cos 8t$
9. $F(p) = \frac{p}{(p + 1)(p^2 + 4p + 5)}$
10. $x'' - 3x' + 2x = 12e^{3t}$, $x(0) = 2$, $x'(0) = 6$
11. $\begin{cases} x' = 3x + 5y + 2, \\ y' = 3x + y + 1, \end{cases}$
 $x(0) = 0$, $y(0) = 2$

Вариант 5.

1. $z = -8i$
2. $z = 4\sqrt{2} - 4\sqrt{2}i$
3. $z = \frac{3 - 5i}{4i + 1}$
4. $f(z) = \frac{\sin \pi z}{(z^2 - 1)^2}$
5. $f(z) = \frac{1}{z^2 - 2z + 5}$
6. $\oint_{|z-1|=1} \frac{e^{2z}}{z^3 - 1} dz$
7. $\oint_{|z-6|=1} \frac{\sin^3 z + 2}{z^2 - 4\pi^2} dz$
8. $f(t) = 2e^{-3t} \sin 4t - (4t^2 + 2t)e^{-t}$
9. $F(p) = \frac{3p - 2}{(p - 1)(p^2 - 6p + 10)}$
11. $\begin{cases} x' = 2x - 2y, \\ y' = -4x, \end{cases}$
10. $x'' + 4x = 8 \sin 2t, x(0) = 3, x'(0) = -1$
- $x(0) = 3, y(0) = 1$

Вариант 6.

1. $z = 6i$
2. $z = -2\sqrt{3} - 2i$
3. $z = \frac{-3 - 7i}{2i - 1}$
4. $f(z) = \frac{\sin z}{z^3(1 - \cos z)}$
5. $f(z) = \frac{z + 1}{z^2}$
6. $\oint_{|z-i|=2} \frac{z^3}{z^4 + 1} dz$
7. $\oint_{|z+1|=1/2} \frac{\operatorname{tg} z + 2}{4z^2 + \pi z} dz$
8. $f(t) = 3t^2 + t - 2 + 3e^{-7t} \cos 2t$
9. $F(p) = \frac{1}{p^5 + p^3}$
11. $\begin{cases} x' = x + 2y, \\ y' = 2x + y + 1, \end{cases}$
10. $2x'' + 5x' = 29 \cos t, x(0) = -1, x'(0) = 0$
- $x(0) = 0, y(0) = 5$

Вариант 7.

1. $z = 3$
2. $z = 2\sqrt{3}i - 6$
3. $z = \frac{7 - 2i}{3i + 5}$
4. $f(z) = \frac{\sin \pi z}{(z - 1)^3}$
5. $f(z) = \frac{z^2}{(z^2 + 1)(z - 1)}$
6. $\oint_{|z|=4} \frac{1 - \cos z}{z^3 - \frac{\pi}{2}z^2} dz$
7. $\oint_{|z-1/2|=1} \frac{2 + \sin z}{z(z + 2i)} dz$
8. $f(t) = (t^2 + 2)e^{2t} - e^{-3t} \cos 2t$
9. $F(p) = \frac{p}{(p - 1)(p^2 - 4p + 4)}$
11. $\begin{cases} x' = 2x + 5y, \\ y' = x - 2y + 2, \end{cases}$
10. $x'' - 2x' - 3x = 2t, x(0) = 1, x'(0) = 1$
- $x(0) = 1, y(0) = 1$

Вариант 8.

1. $z = -4i$
2. $z = -3 + 3i$
3. $z = \frac{4i - 3}{6i - 5}$
4. $f(z) = z^2 \sin \frac{1}{z}$
5. $f(z) = \frac{z}{(z-1)(z-3)}$
6. $\oint_{|z-1-i|=\sqrt{2}} \frac{dz}{(z-1)^2(z^2+1)}$
7. $\oint_{|z-2|=3} \frac{\cos^2 z + 1}{z^2 - \pi^2} dz$
8. $f(t) = 8e^{-2t} \sin 3t + e^{2t} \cos 8t$
9. $F(p) = \frac{1}{p^3 + p^2 + 2p + 2}$
11. $\begin{cases} x' = -4x + y, \\ y' = -2x - y, \end{cases}$
10. $x'' - x' = t^2, x(0) = 0, x'(0) = 1$
- $x(0) = 2, y(0) = 3$

Вариант 9.

1. $z = -7$
2. $z = -2i + 2\sqrt{3}$
3. $z = \frac{3 + 4i}{2i + 3}$
4. $f(z) = \frac{ze^z}{\sin z}$
5. $f(z) = \frac{z-1}{z^2+4}$
6. $\oint_{|z|=10} \frac{\sin^3 z + 2}{z^2 + 4\pi^2} dz$
7. $\oint_{|z-1|=2} \frac{3z^4 - 2z^3 + 5}{z^4} dz$
8. $f(t) = e^{-4t} \sin 3t \cos 2t + t^2 \sin t$
9. $F(p) = \frac{p}{p^4 - 1}$
11. $\begin{cases} x' = -7x + y, \\ y' = -2x - 5y, \end{cases}$
10. $x'' + 2x' + x = \cos t, x(0) = 0, x'(0) = 0$
- $x(0) = 1, y(0) = 1$

Вариант 10.

1. $z = 3i$
2. $z = 12i - 4\sqrt{3}$
3. $z = \frac{5i - 1}{3i + 8}$
4. $f(z) = \operatorname{ctg}^2 z$
5. $f(z) = \frac{1}{z(z^2+1)}$
6. $\oint_{|z|=2} \frac{\sin^2 z}{z \cos z} dz$
7. $\oint_{|z+1/2|=3} \frac{z^3 - 3z^2 + 1}{2z^4} dz$
8. $f(t) = 5e^{3t} \cos 3t \cos 4t + 1 + t^2 e^{3t}$
9. $F(p) = \frac{4p^2 + 16p - 8}{p^3 - 4p}$
11. $\begin{cases} x' = -x - 2y + 1, \\ y' = -3x + y, \end{cases}$
10. $x'' + x' = t^2 + 2t, x(0) = 4, x'(0) = -2$
- $x(0) = 2, y(0) = 0$

Вариант 11.

1. $z = 4$
2. $z = 3 - 3\sqrt{3}i$
3. $z = \frac{4 - 6i}{2i - 3}$
4. $f(z) = \frac{\sin 3z}{z(1 - \cos z)}$
5. $f(z) = \frac{z^2 + 4}{(z - 1)^3}$
6. $\oint_{|z-1|=1} \frac{dz}{z^4 + 1}$
7. $\oint_{|z-3|=1/2} \frac{e^z}{\sin z} dz$
8. $f(t) = t(e^t + \operatorname{sh} t) - 2 \sin^2 2t$
9. $F(p) = \frac{p + 3}{p^3 + p^2 - 2p}$
11. $\begin{cases} x' = -y, \\ y' = 2x + 2y, \end{cases}$
10. $x'' + 9x = \cos 3t, x(0) = 1, x'(0) = 0$
- $x(0) = 1, y(0) = 1$

Вариант 12.

1. $z = -6i$
2. $z = -15i + 5\sqrt{3}$
3. $z = \frac{3i + 1}{4 - 2i}$
4. $f(z) = \frac{e^z - 1}{\sin \pi z}$
5. $f(z) = \frac{1}{z^3 - 1}$
6. $\oint_{|z|=2} \frac{z^2}{\sin^2 z \cos z} dz$
7. $\oint_{|z-i|=3/2} \frac{dz}{z(z^2 + 4)}$
8. $f(t) = \operatorname{sh} t \cos 2t \sin 3t + t^3 e^{-3t}$
9. $F(p) = \frac{1}{p^3 + 8}$
11. $\begin{cases} x' = x + 4y, \\ y' = 2x - y + 9, \end{cases}$
10. $x'' + 3x' + 2x = 1 + t + t^2, x(0) = 0, x'(0) = 1$
- $x(0) = 1, y(0) = 0$

Вариант 13.

1. $z = 2$
2. $z = 5\sqrt{2}i - 5\sqrt{2}$
3. $z = \frac{4i - 6}{6i + 5}$
4. $f(z) = \frac{1}{\cos z}$
5. $f(z) = \frac{z^2}{(z^2 + 1)^2}$
6. $\oint_{|z|=2} \frac{dz}{z(z^2 + 1)}$
7. $\oint_{|z-1|=3} \frac{1 - \sin \frac{1}{z}}{z} dz$
8. $f(t) = -\frac{t}{2} \sin 2t - e^{-3t} \cos t$
9. $F(p) = \frac{p^2 - 3}{p^4 + 5p^2 + 6}$
11. $\begin{cases} x' = -2x + 5y + 1, \\ y' = x + 2y + 1, \end{cases}$
10. $x'' - x = \cos 3t, x(0) = 1, x'(0) = 1$
- $x(0) = 0, y(0) = 2$

Вариант 14.

1. $z = -5i$
2. $z = 6i - 6\sqrt{3}$
3. $z = \frac{3 - 2i}{-6 - 5i}$
4. $f(z) = \frac{1}{\cos^2 z}$
5. $f(z) = \frac{e^z - 1}{z^2 + z}$
6. $\oint_{|z-1-i|=5/4} \frac{2}{z^2(z-1)} dz$
7. $\oint_{|z|=1} \frac{z^3 - i}{\sin 2z \cdot (z - \pi)} dz$
8. $f(t) = 3t^4 e^{2t} + e^{-t} \sin 8t$
9. $F(p) = \frac{2p^2 - 3p + 1}{p^3 + 1}$
11. $\begin{cases} x' = 3x + y, \\ y' = -5x - 3y + 2, \end{cases}$
 $x(0) = 2, y(0) = 0$
10. $x'' + x' + x = 7e^{2t}, x(0) = 1, x'(0) = 4$

Вариант 15.

1. $z = 7$
2. $z = 5i + 5\sqrt{3}$
3. $z = \frac{8i - 3}{4i + 2}$
4. $f(z) = \frac{z + \pi}{z \sin z}$
5. $f(z) = \frac{z}{z^2 + 4z + 5}$
6. $\oint_{|z|=1} \frac{2 + \sin z}{z(z + 2i)} dz$
7. $\oint_{|z+2|=4} \frac{e^{3z} - 1}{z^3} dz$
8. $f(t) = 2t \cos 3t - t^3 e^{4t} + 1 - t^2$
9. $F(p) = \frac{p^2}{p^4 - 81}$
11. $\begin{cases} x' = -3x - 4y + 1, \\ y' = 2x + 3y, \end{cases}$
 $x(0) = 0, y(0) = 2$
10. $x'' - 9x = \sin t - \cos t, x(0) = -3, x'(0) = 2$

Вариант 16.

1. $z = -5$
2. $z = -4\sqrt{3}i - 12$
3. $z = \frac{7 - 4i}{6i + 1}$
4. $f(z) = \frac{1}{\sin^2 z}$
5. $f(z) = \frac{\sin 2z}{(z^2 + 1)^2}$
6. $\oint_{|z-3/2|=2} \frac{z(\sin z + 2)}{\sin z} dz$
7. $\oint_{|z|=1} \frac{\cos z^2 - 1}{z^3} dz$
8. $f(t) = \operatorname{ch} 3t \sin^2 t - t^{10} e^t$
9. $F(p) = \frac{2p + 3}{p^3 + 4p^2 + 5p}$
11. $\begin{cases} x' = 2x + 8y + 1, \\ y' = 3x + 4y, \end{cases}$
 $x(0) = 2, y(0) = 1$
10. $x'' + x' - 2x = -2t - 2, x(0) = 1, x'(0) = 1$

Вариант 17.

1. $z = -2i$
2. $z = 4 + 4\sqrt{3}i$
3. $z = \frac{5i + 1}{2 + 3i}$
4. $f(z) = \frac{1}{e^z - 1}$
5. $f(z) = \frac{e^{2z}}{z^2(z - 1)}$
6. $\oint_{|z-3|=1/2} \frac{e^z}{\sin z} dz$
7. $\oint_{|z|=1/3} \frac{z^4 + 2z^2 + 3}{2z^6} dz$
8. $f(t) = 2 - 3t^2 + t \cos 5t + e^{-t} \sin 3t$
9. $F(p) = \frac{p^2 - p + 1}{p^4 + 2p^2 - 3}$
11. $\begin{cases} x' = 2x + 2y + 2, \\ y' = 4y + 1, \end{cases}$
10. $x'' + 2x' = 2 + e^t, x(0) = 1, x'(0) = 2$
- $x(0) = 0, y(0) = 1$

Вариант 18.

1. $z = 6$
2. $z = 3\sqrt{2} + 3\sqrt{2}i$
3. $z = \frac{6i - 8}{3 - 2i}$
4. $f(z) = \frac{\sin z^2}{z(z^2 + 1)}$
5. $f(z) = \frac{z + 1}{(z - 1)^2(z + 3)}$
6. $\oint_{|z-1/2|=1} \frac{e^z + 1}{z(z + 1)} dz$
7. $\oint_{|z|=1} \frac{z^2 e^{1/z^2} - 1}{z} dz$
8. $f(t) = \operatorname{sh} 4t \cos^2 3t - t \cos 5t$
9. $F(p) = \frac{p^3 - 6}{p^4 + 6p^2 + 8}$
11. $\begin{cases} x' = x + y, \\ y' = 4x + y + 1, \end{cases}$
10. $2x'' - x' = \sin 3t, x(0) = 2, x'(0) = 1$
- $x(0) = 1, y(0) = 0$

Вариант 19.

1. $z = -7i$
2. $z = -5 + 5\sqrt{3}i$
3. $z = \frac{6i - 1}{3 - 8i}$
4. $f(z) = \frac{z^2 + 1}{(z - i)^2(z^2 + 4)}$
5. $f(z) = \frac{z^4}{(z^2 + 1)^2}$
6. $\oint_{|z-1/2|=1} \frac{z(z - i)}{\sin \pi z} dz$
7. $\oint_{|z|=1/3} \frac{1 - 2z^4 + 3z^5}{z^4} dz$
8. $f(t) = t^2 e^t + 4e^{2t} \cos 5t$
9. $F(p) = \frac{p + 5}{(p - 1)(p^2 - 2p + 5)}$
11. $\begin{cases} x' = x + 4y + 1, \\ y' = 2x + 3y, \end{cases}$
10. $x'' + 2x' = \sin \frac{t}{2}, x(0) = -2, x'(0) = 4$
- $x(0) = 0, y(0) = 1$

Вариант 20.

1. $z = -2$
2. $z = -2\sqrt{3} - 6i$
3. $z = \frac{7i + 4}{2i + 5}$
4. $f(z) = \frac{\cos\left(\frac{\pi}{2}z\right)}{z^2 - 1}$
5. $f(z) = \frac{z^6}{(z - 1)^4}$
6. $\oint_{|z-3|=10} \frac{\sin 3z + 2}{z^2(z - \pi)} dz$
7. $\oint_{|z|=1/2} \frac{z^5 - 3z^2 + 5z}{z^4} dz$
8. $f(t) = t^2 \cos t - \frac{1}{2}t^4 e^{-2t} + e^t \sin 3t$
9. $F(p) = \frac{3p + 2}{(p + 1)(p^2 + 4p + 5)}$
11. $\begin{cases} x' = x + 3y + 3, \\ y' = x - y + 1, \end{cases}$
10. $x'' + x = \operatorname{sh} t, x(0) = 2, x'(0) = 1$
- $x(0) = 0, y(0) = 1$

Вариант 21.

1. $z = 4i$
2. $z = -3\sqrt{3} + 3i$
3. $z = \frac{4 - 8i}{3i - 1}$
4. $f(z) = \frac{1}{z^2} + \sin \frac{1}{z^2}$
5. $f(z) = \frac{z^5}{z^2 - 1}$
6. $\oint_{|z|=3} \frac{\cos^2 z + 3}{2z^2 + \pi z} dz$
7. $\oint_{|z-1/5|=2} \frac{1 - z^2 + 3z^4}{2z^3} dz$
8. $f(t) = \operatorname{sh} 2t \sin^2 3t - 3 + t \sin t$
9. $F(p) = \frac{2p^2 - 4p + 8}{(p - 2)^2(p^2 + 4)}$
11. $\begin{cases} x' = -x + 3y + 2, \\ y' = x + y + 1, \end{cases}$
10. $x'' + 4x' + 20x = e^{-2t}, x(0) = 0, x'(0) = 1$
- $x(0) = 0, y(0) = 1$

Вариант 22.

1. $z = 5$
2. $z = -2i - 2$
3. $z = \frac{5i + 7}{6i - 2}$
4. $f(z) = \frac{\sin \pi z}{z(z - 1)^2}$
5. $f(z) = \frac{e^z}{(z - 3)^2(z + 5)}$
6. $\oint_{|z|=3} \frac{dz}{z(z^2 + 1)}$
7. $\oint_{|z|=1} \frac{e^{zi} + 2}{\sin 3zi} dz$
8. $f(t) = 1 + 2t^5 - \operatorname{sh} t \cos 4t$
9. $F(p) = \frac{1}{p(p^3 + 1)}$
11. $\begin{cases} x' = x + 3y, \\ y' = x - y, \end{cases}$
10. $x'' - 3x' + 2x = e^t, x(0) = 1, x'(0) = 0$
- $x(0) = 1, y(0) = 0$

Вариант 23.

1. $z = -3$
2. $z = 6 - 2\sqrt{3}i$
3. $z = \frac{3i - 5}{4 + 3i}$
4. $f(z) = \frac{\cos \pi z}{(z-1)(z^2+1)}$
5. $f(z) = \frac{\cos 4z}{(z-i)^3}$
6. $\oint_{|z+1|=2} \frac{\sin^2 z - 3}{z^2 + 2\pi z} dz$
7. $\oint_{|z|=1/3} \frac{1 - 2z + 3z^2 + 4z^3}{2z^2} dz$
8. $f(t) = 2e^{-2t} \sin 5t - t + t^3 e^t$
9. $F(p) = \frac{5}{(p-1)(p^2+4p+5)}$
11. $\begin{cases} x' = 2x + 3y + 1, \\ y' = 4x - 2y, \end{cases}$
10. $2x'' + 3x' + x = 3e^t, x(0) = 0, x'(0) = 1$
- $x(0) = -1, y(0) = 0$

Вариант 24.

1. $z = 5i$
2. $z = 5 - 5i$
3. $z = \frac{2i + 8}{-3 - 8i}$
4. $f(z) = \frac{\cos(\frac{\pi}{2}z)}{(z-1)^3}$
5. $f(z) = \frac{z}{(z-5)^3}$
6. $\oint_{|z|=\pi/2} \frac{z^2 + z + 3}{\sin z \cdot (\pi + z)} dz$
7. $\oint_{|z-i|=3} \frac{e^z - \sin z}{z^2} dz$
8. $f(t) = e^{3t} \cos t \cos 3t + \frac{t}{2} - 2 + te^{-t}$
9. $F(p) = \frac{1}{(p-2)(p^2+2p+3)}$
11. $\begin{cases} x' = 3y + 2, \\ y' = x + 2y, \end{cases}$
10. $x'' + 4x = \sin 2t, x(0) = 0, x'(0) = 1$
- $x(0) = -1, y(0) = 1$

Вариант 25.

1. $z = -6$
2. $z = 9i + 3\sqrt{3}$
3. $z = \frac{-4i - 1}{3i - 2}$
4. $f(z) = \frac{2z - \sin 2z}{z^2(z^2+1)}$
5. $f(z) = \frac{z^2 + z - 1}{z^2(z-1)}$
6. $\oint_{|z-1|=3} \frac{z(z+\pi)}{\sin z} dz$
7. $\oint_{|z|=1/3} \frac{4z^5 - 3z^3 - 1}{z^6} dz$
8. $f(t) = 5t \cos 2t - e^{2t} t^3 + e^{-t} \sin t$
9. $F(p) = \frac{2p+1}{(p+1)(p^2+2p+3)}$
11. $\begin{cases} x' = -2x + y, \\ y' = 3x, \end{cases}$
10. $x'' + x' - 2x = e^{-t}, x(0) = -1, x'(0) = 0$
- $x(0) = 0, y(0) = 1$

П Р И Л О Ж Е Н И Я

Приложение 1.

Таблица производных

$$(c)' = 0 \quad (c - \text{число})$$

$$x' = 1$$

$$(x^2)' = 2x$$

$$(x^n)' = n x^{n-1}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$(e^x)' = e^x$$

$$(\ln x)' = \frac{1}{x}$$

$$(\sin x)' = \cos x$$

$$(a^x)' = a^x \ln a$$

$$(\log_a x)' = \frac{1}{x \ln a}$$

$$(\cos x)' = -\sin x$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$$

$$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$$

$$(\operatorname{sh} x)' = \operatorname{ch} x$$

$$(\operatorname{ch} x)' = \operatorname{sh} x$$

$$(u+v)' = u' + v'$$

$$(u-v)' = u' - v'$$

$$(cu)' = cu' \quad (c - \text{число})$$

$$(uv)' = u'v + uv'$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

Таблица дифференциалов

$$d(f(x)) = (f(x))' dx$$

$$d(a) = 0 \quad (a - \text{число})$$

$$dx = d(x + a)$$

$$dx = d(x - a)$$

$$dx = -d(-x)$$

$$dx = \frac{1}{b} d(bx)$$

$$dx = b d\left(\frac{x}{b}\right)$$

$$x^n dx = \frac{1}{n+1} d(x^{n+1})$$

$$\frac{dx}{x} = d(\ln x)$$

$$dx = \frac{1}{a} d(ax + b)$$

$$x dx = \frac{1}{2} d(x^2)$$

$$\frac{dx}{x^2} = -d\left(\frac{1}{x}\right)$$

$$\frac{dx}{\sqrt{x}} = 2 d(\sqrt{x})$$

$$e^x dx = d(e^x)$$

$$\cos x dx = d(\sin x)$$

$$\frac{dx}{\cos^2 x} = d(\operatorname{tg} x)$$

$$a^x dx = \frac{1}{\ln a} d(a^x)$$

$$\sin x dx = -d(\cos x)$$

$$\frac{dx}{\sin^2 x} = -d(\operatorname{ctg} x)$$

$$\frac{dx}{\sqrt{1-x^2}} = d(\arcsin x)$$

$$\frac{dx}{1+x^2} = d(\operatorname{arctg} x)$$

$$\operatorname{ch} x dx = d(\operatorname{sh} x)$$

$$\frac{dx}{\sqrt{1-x^2}} = -d(\arccos x)$$

$$\frac{dx}{1+x^2} = -d(\operatorname{arcctg} x)$$

$$\operatorname{sh} x dx = d(\operatorname{ch} x)$$

Таблица интегралов

$$\int 0 dx = C$$

$$\int dx = \int 1 dx = x + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int x dx = \frac{x^2}{2} + C$$

$$\int \frac{dx}{x^2} = -\frac{1}{x} + C$$

$$\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$\int \frac{dx}{\sqrt{x^2 + k}} = \ln|x + \sqrt{x^2 + k}| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$$

$$\int F'(x) dx = \int d(F(x)) = F(x) + C$$

$$\int u dv = uv - \int v du$$

$$\int_a^b u dv = uv \Big|_a^b - \int_a^b v du$$

Таблица разложений в степенные ряды

$$\begin{aligned}
 e^z &= \sum_{n=0}^{\infty} \frac{z^n}{n!} = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots + \frac{z^n}{n!} + \dots, \quad |z| < \infty, \\
 \sin z &= \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!} = \\
 &= z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots + (-1)^n \frac{z^{2n+1}}{(2n+1)!} + \dots, \quad |z| < \infty, \\
 \cos z &= \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!} = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots + (-1)^n \frac{z^{2n}}{(2n)!} + \dots, \quad |z| < \infty, \\
 \operatorname{sh} z &= \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!} = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \dots + \frac{z^{2n+1}}{(2n+1)!} + \dots, \quad |z| < \infty, \\
 \operatorname{ch} z &= \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} = 1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \dots + \frac{z^{2n}}{(2n)!} + \dots, \quad |z| < \infty, \\
 (1+z)^m &= 1 + \sum_{n=1}^{\infty} \frac{m(m-1)(m-2)\dots(m-n+1)}{n!} z^n = \\
 &= 1 + mz + \frac{m(m-1)}{2!} z^2 + \frac{m(m-1)(m-2)}{3!} z^3 + \dots + \\
 &\quad + \frac{m(m-1)(m-2)\dots(m-n+1)}{n!} z^n + \dots, \quad m \in \mathbb{Z}, \quad |z| < 1.
 \end{aligned}$$

Приведём некоторые частные случаи последней формулы.

$$\begin{aligned}
 \frac{1}{1+z} &= 1 - z + z^2 - z^3 + \dots + (-1)^n z^n + \dots, \quad |z| < 1, \\
 \frac{1}{1-z} &= 1 + z + z^2 + z^3 + \dots + z^n + \dots, \quad |z| < 1.
 \end{aligned}$$

Таблица изображений и оригиналов

Изображение $F(p)$	Оригинал $f(t)$
$\frac{1}{p}$	1
$\frac{a}{p^2 + a^2}$	$\sin at$
$\frac{p}{p^2 + a^2}$	$\cos at$
$\frac{1}{p + b}$	e^{-bt}
$\frac{a}{p^2 - a^2}$	$\operatorname{sh} at$
$\frac{p}{p^2 - a^2}$	$\operatorname{ch} at$
$\frac{a}{(p + b)^2 + a^2}$	$e^{-bt} \sin at$
$\frac{p + b}{(p + b)^2 + a^2}$	$e^{-bt} \cos at$
$\frac{n!}{p^{n+1}}$	t^n
$\frac{2pa}{(p^2 + a^2)^2}$	$t \sin at$
$\frac{p^2 - a^2}{(p^2 + a^2)^2}$	$t \cos at$
$\frac{1}{(p + b)^2}$	te^{-bt}
$\frac{1}{(p^2 + a^2)^2}$	$\frac{1}{2a^3}(\sin at - at \cos at)$
$\frac{n!}{(p - b)^{n+1}}$	$t^n e^{bt}$
$(-1)^n \frac{d^n}{dp^n} F(p)$	$t^n f(t)$
$F_1(p) \cdot F_2(p)$	$\int_0^t f_1(\tau) f_2(t - \tau) d\tau$

Везде в таблице $n \in \mathbb{N}$, $a \in \mathbb{R}$, $b \in \mathbb{R}$.

Некоторые формулы тригонометрии

$$\sin^2 x + \cos^2 x = 1$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x} \qquad \operatorname{ctg} x = \frac{\cos x}{\sin x}$$

$$\cos 2x = \cos^2 x - \sin^2 x \qquad \cos 2x = 2 \cos^2 x - 1 \qquad \cos 2x = 1 - 2 \sin^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos^2 x = \frac{1 + \cos 2x}{2} \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\sin x \cos y = \frac{1}{2} (\sin(x - y) + \sin(x + y))$$

$$\cos x \cos y = \frac{1}{2} (\cos(x - y) + \cos(x + y))$$

$$\sin x \sin y = \frac{1}{2} (\cos(x - y) - \cos(x + y))$$

$$\operatorname{sh} x = \frac{e^x - e^{-x}}{2} \text{ — гиперболический синус}$$

$$\operatorname{ch} x = \frac{e^x + e^{-x}}{2} \text{ — гиперболический косинус}$$

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